frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately $2T$. This machine has $MA \approx 2$. (b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of $4T$, so that it has $MA \approx 4$. Effectively, four cables are pulling on the system of interest.

9.6 Forces and Torques in Muscles and Joints

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 9.25 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in Figure 9.25.

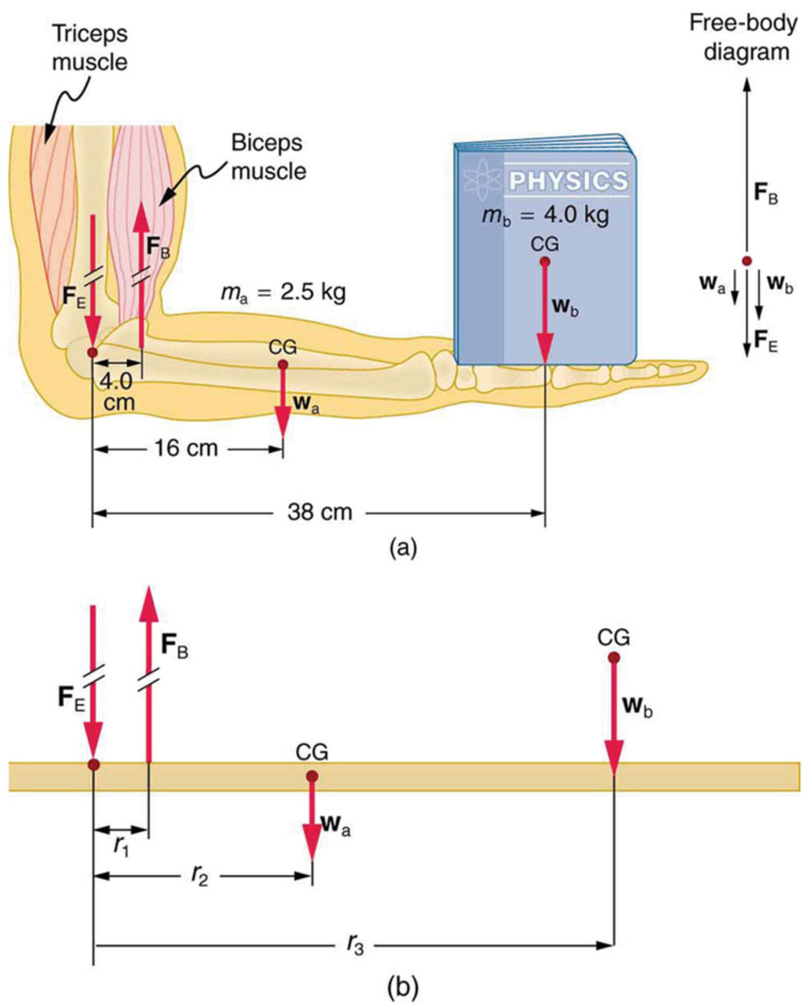

FIGURE 9.25 (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F}_B$ to support the weight of the forearm

and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in Example 9.4.

EXAMPLE 9.4

Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 9.25, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_{B}; that of the elbow joint is F_{E}; that of the weights of the forearm is w_{a}, and its load is w_{b}. Two of these are unknown (F_{B} and F_{E}), so that the first condition for equilibrium cannot by itself yield F_{B}. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_{E} is zero, and the only unknown becomes F_{B}.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium $(\text{net } \boldsymbol{\tau} = 0)$ becomes

$$r_2 w_{\text{a}} + r_3 w_{\text{b}} = r_1 F_{\text{B}}. \tag{9.35}$$

Note that $\sin \theta = 1$ for all forces, since $\theta = 90^{\circ}$ for all forces. This equation can easily be solved for F_{B} in terms of known quantities, yielding

$$F_{\text{B}} = \frac{r_2 w_{\text{a}} + r_3 w_{\text{b}}}{r_1}. \tag{9.36}$$

Entering the known values gives

$$F_{\text{B}} = \frac{(0.160 \text{ m})(2.50 \text{ kg})\left(9.80 \text{ m/s}^2\right)+(0.380 \text{ m})(4.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)}{0.0400 \text{ m}} \tag{9.37}$$

which yields

$$F_{\text{B}} = 470 \text{ N}. \tag{9.38}$$

Now, the combined weight of the arm and its load is $(6.50 \text{ kg})\left(9.80 \text{ m/s}^2\right)= 63.7 \text{ N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_{\text{B}}}{w_{\text{a}} + w_{\text{b}}} = \frac{470}{63.7} = 7.38. \tag{9.39}$$

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_{E} exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of F_{E} is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, $470 \text{ N} - 407 \text{ N} = 63 \text{ N}$, approximately equal to the weight supported.) Forces in muscles and