# Potential Energy

## Recap

Potential Energy
- ★ Stored energy
- ★ $PE_g$=mgh
- ★ $PE_{Spring}$= $\frac{1}{2}kx^2$
- ★ Units: Joules (J)

Conservation of Energy
- ★ $\Delta Kenetic\ Energy\ +\ \Delta Potential = 0\ =\ Constant$
  - ○ $KE_i+PE_i= KE_f+PE_f$
    - ■ $\frac{1}{2}mv_i + mgh_i = \frac{1}{2}mv_f + mgh_f$
- ★ If going down a hill and starting from rest and hitting the bottom then
  - ○ $PE_{top}=KE_{bottom}$
    - ■ This is because the initial speed is 0 and the final height is also 0, canceling out kinetic energy at the top and potential energy at the bottom

Practice

(a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about 7 × $10^9$ kg and its center of mass is 36.5 m above the surrounding ground? [Video on how to solve]

ⓐ $PE_g = mgh$ $h = 36.5$ $g = 9.8$ $m = 7\times10^9 kg$

$= (7\times10^9)\cdot 9.8 \cdot 36.5 = 2.5039\times^{12} \approx 3\times10^{12}$

A 5.00 × $10^5$-kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring? [Video on how to solve]

$m = 5\times10^5 kg$ $v_0 = .5\,m/s$ $v_f = 0\,m/s$ $x = .4\,m$ $k = ?$

$PE_s + K = 0$

$PE_s = \frac{1}{2}kx^2$

$-K = \frac{1}{2}kx^2$

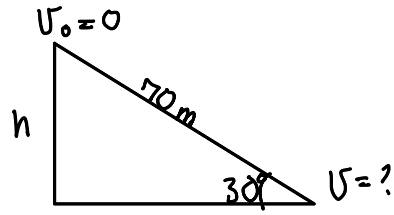
$2\cdot -(\frac{1}{2}mv^2) = \frac{1}{2}kx^2 \cdot 2$

$-(m\Delta v^2) = kx^2$

$k = \frac{-(m\Delta v^2)}{x^2}$

$k = \frac{-(5\times10^5 kg(-(.5\,m/s)^2))}{.4\,m} = 7.81\times10^5 \frac{N}{m}$

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who travels 70.0 m along a 30º slope, neglecting friction: Video on how to solve




$v_i = 0$ $h_f = 0$ $\theta = 30°$

(a) Starting from rest.

$\cancel{K_i} + PE_i = K_f + \cancel{PE_f}$

$2 \cdot \cancel{m} g h_0 = \frac{1}{2} \cancel{m} v_f^2 \cdot 2$

$2 g h_0 = v_f^2$

$v_f = \sqrt{2 g h_0}$

$v_f = \sqrt{2\ 9.8\ (70 \sin 30°)} = 26.2\ m/s$

time

$x = \frac{1}{2}(\cancel{v_0} + v_f)t$

$t = \frac{2x}{v_f} = \frac{2 \cdot 70\,m}{26.2\,m/s}$

$\boxed{t = 5.34\ secs}$

$\sin\theta = \frac{h_0}{x}$

$h_0 = x \sin\theta =$

$h_0 = 70 \sin 30$

(b) Starting with an initial speed of 2.50 m/s.

$K_i + PE_i = K_f + \cancel{PE_f}$ $v_0 = 2.50\ m/s$

$2 \cdot \frac{1}{2} \cancel{m} v_0^2 + \cancel{m} g h_0 = \frac{1}{2} \cancel{m} v_f^2 \cdot 2$

$v_0^2 + g h_0 = v_f^2$

$v_f = \sqrt{v_0^2 + g h_0}$

$v_f = \sqrt{2.5^2\,m/s + 9.8\,m/s^2\,(70\,m \sin 30°)}$

$v_f = 26.25$

Time

$2 \cdot x = \frac{1}{2}(v_i + v_f)t \cdot 2$

$(v_i + v_f)$

$t = \frac{2 \cdot x}{(v_0 + v_f)} = \frac{2 \cdot 70\,m}{(2.5\,m/s + 26.25\,m/s)}$

$t = 4.87\ sec$

(c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

The 70.0-kg swimmer in Figure 7.41 starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m-long stroke. [Video on how to solve]

$m = 70\,kg \qquad v_0 = 1.25\,m/s \qquad F_{avg} = 80\,N \qquad x = 1.80\,m$

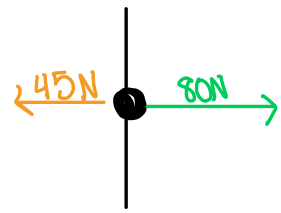
(a) What is his initial acceleration if water resistance is 45.0 N?

$F_w = -45\,N \qquad \Sigma F = \text{all total forces}$

$\Sigma F_x = ma_x$

$F_{Avg} + F_w = ma_x$

$a_x = \frac{F_{Avg} + F_w}{m} = \frac{80N + (-45N)}{70} = .500\,m/s^2$

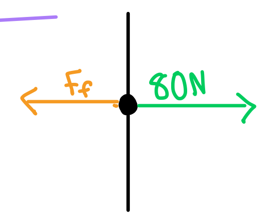



(b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s?

$v_f = 2.50\,m/s \qquad t = 5s$

$a_x = \frac{\Delta v}{t} = \frac{2.5\,m/s - 1.25\,m/s}{5\,s}$

$a_x = .25\,m/s^2$




$\Sigma F_x = ma_x$

$F_{avg} + (-F_r) = ma_x$

$-F_{avg} \qquad -F_{avg}$

$-F_r = ma_x - F_{avg}$

$F_r = -ma_x + F_{avg} = -70\,kg \cdot .25\,m/s^2 + 80 = 62.5\,N$

~~(c) Discuss whether water resistance seems to increase linearly with velocity.~~

A 75.0-kg cross-country skier is climbing a 3.0º slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. [Video on how to solve]

$m = 75 kg \quad \theta = 3 \quad F_r = 25.0 N \quad v = 2 m/s$

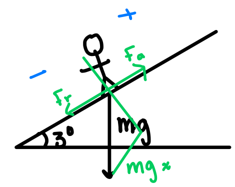
(a) Find his power output for work done against the gravitational force and air resistance.

$P = F \cdot v$

$P = F_a \cdot v$

$P = (25 N + (75 kg \cdot 9.8 m/s^2 \cdot \sin 3) \cdot 2 m/s$

$P = 127 W$




$\Sigma F_x = 0$

$\Sigma F = F_a - W_x - F_r =$

$F_a = F_r + W_x =$

$F_a = 25N + 75 kg \cdot 9.8 m/s^2 \sin 3°$

(b) What average force does he exert backward on the snow to accomplish this?

$F_a = 25 N + 75 kg (9.8 m/s^2) \sin 3$

$F = 63.5 N$

(c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

$\Sigma F_x = ma$

$F_a - F_r = ma$

$a = \frac{F_a - F_r}{m}$

$a = \frac{63.5 N - 25 N}{75 kg} = .513 m/s^2$

$a = \frac{\Delta v}{t}$

$t = \frac{\Delta v}{a}$

$t = \frac{v_f - v_i}{a}$

$t = \frac{10.0 - 2.0}{.513 m/s^2}$

$t = 15.6 secs$